Some aspects of modern lattice problems

Abstract:

In this talk, I will discuss modern research questions related to Euclidean lattices. These questions include the energy minimization problem, the lattice polarization problem, the problem of finding least-distortion embeddings of flat tori into Hilbert spaces, and the lattice coloring problem.

These problems have seen increased interest in recent years, and are sometimes closely related to classical lattice problems, such as the sphere packing problem, the sphere covering problem, and the closest vector problem. Although these modern problems have not been around for as long as the classical ones, they have established themselves as promising areas of research in extremal discrete geometry.

I will place special emphasis on two modern extensions of classical problems: the energy minimization problem and the lattice polarization problem. The former is an extension of the sphere packing problem and the latter is an extension of the sphere covering problem. While these problems share characteristics with their classical counterparts, they exhibit new and sometimes unintuitive behavior. To illustrate, I will discuss expected and unexpected geometric phenomena and highlight the similarities and differences between the modern problems and their classical counterparts.